TIME AND FREQUENCY:
Theory and Fundamentals

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Given:

$$A_{ptp}=0.316\ V\left(\text{i.e., } \frac{1}{\sqrt{10}}V\right) \quad (8.D.1)$$

$$v=100\ \text{nV Hz}^{-1/2}\ @\ f=20\ \text{Hz}, \quad (8.D.2)$$

from a pair of equally noisy signals.

$$\mathscr{L}(20\ \text{Hz})=\left(\frac{v}{A_{ptp}}\right)^2 \quad (8.D.3)$$

$$=\left(\frac{100\ \text{nV Hz}^{-1/2}}{0.316\ \text{V}}\right)^2=\left(\frac{10^{-7}}{\sqrt{10^{-1}}}\right)\ \text{Hz}^{-1}=\frac{10^{-14}}{10^{-1}}\ \text{Hz}^{-1}$$

$$=10^{-13}\ \text{Hz}^{-1}=-130\ \text{dB},$$

or using logarithms:

$$\mathscr{L}(20\ \text{Hz})=20\ \log_{10}\left(\frac{v}{A_{ptp}}\right) \quad (8.D.4)$$

$$=20\ \log_{10}\frac{(10^{-7}\ \text{V}\cdot\text{Hz}^{-1/2})}{(10^{-1/2}\ \text{V})}=20(-7+0.5)$$

$$=-130\ \text{dB}.$$

If the phase noise follows flicker law, at $f=1$ Hz it is 20 times worse (or 13 dB greater); that is

$$\mathscr{L}(1\text{Hz})=-130\ \text{dB}+13\ \text{dB}=-117\ \text{dB}. \quad (8.D.5)$$

ANNEX 8.E

A SAMPLE CALCULATION OF ALLAN VARIANCE, $\sigma_y^2(\tau)$

$$\sigma_y^2(\tau)\equiv\langle\sigma_y^2(N=2,\ T=\tau,\ \tau)\rangle=\left\langle\frac{(\bar{y}_{k+1}-\bar{y}_k)^2}{2}\right\rangle$$

$$\approx\frac{1}{2(M-1)}\sum_{k=1}^{M-1}(\bar{y}_{k+1}-\bar{y}_k)^2. \quad (8.E.1)$$

in the example below:

Number of data values available, $M=9$
Number of differences averaged, $M-1=8$
Sampling time interval $\tau=1$s

TABLE 8.E.1. *Sample data tabulation*

Data values ($\bar{y}$)	First differences ($\bar{y}_{k+1}-\bar{y}_k$)	First differences squared ($\bar{y}_{k+1}-\bar{y}_k)^2$
892	—	—
809	−83	6889
823	14	196
798	−25	625
671	−127	16129
644	−27	729
883	239	57121
903	20	400
677	−226	51076
	$\sum_{k=1}^{M-1}(\bar{y}_{k+1}-\bar{y}_k)^2=133165$	

Based on these data:

$$\sigma_y^2(\tau)=\frac{133165}{2(8)}=8322.81, \quad (8.E.2)$$

$$[\sigma_y^2(\tau)]^{1/2}=\sqrt{8322.81}=91.23,\ N=2,\ T=\tau=1\ \text{s}. \quad (8.E.3)$$

In this example, the data values may be understood to be expressed in parts in 10^{12}; the data may have been taken as the counted number of periods, in the time interval τ, of the beat frequency between the oscillator under test and a reference oscillator, **divided by the nominal carrier frequency ν_0, and** multiplied by the factor 10^{12}.

Using the same data as in the above example it is possible to calculate the Allan variance for $\tau=2$ s by averaging pairs of adjacent data values and using these averaged values as new data values to proceed with the calculation as before. Allan variance **values may be obtained for $\tau=3$ s by averaging** three adjacent data values in a similar manner, etc., for larger values of τ.

Ideally the calculation is done via a computer and a large number, M, of data values should be used. (Typically $M=256$ data values are used in the NBS computer program.) The statistical confidence of the calculated Allan variance improves nomially as the square root of the number, M, of data values used [19]. For $M=256$, the confidence of the Allan variance is expected to be approximately $\pm\frac{1}{\sqrt{256}}\times 100$ percent $\approx\pm 7$ percent of its value. The use of $M>>1$ is logically similar to the use of $B_a\cdot\tau_a>>1$ in spectrum analysis measurements, where B_a is the analysis bandwidth (frequency window) of the spectrum analyzer, and τ_a is the post-detection averaging time of the spectrum analyzer.